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Research on Lexicographic Linear Goal Programming Problem Based on LINGO and Column-Dropping Rule

N. R. Neelavathi

Assistant Professor, Department of Mathematics, P.K.R. Arts College for women, Gobichettipalayam, Erode district, Tamilnadu, India

Abstract: **Lexicographic Linear Goal programming within a pre-emptive priority structure including Column-dropping Rule has been one of the useful techniques considered in solving multiple objective problems. The basic ideas to solve goal programming are transforming goal programming into single-objective linear programming. An optimal solution is attained when all the goals are reached as close as possible to their aspiration level, while satisfying a set of constraints. One of the Goal Programming algorithm – the Lexicographic method including Column-dropping Rule and the method of LINGO software are discussed in this paper. Finally goal programming model are applied to the actual management decisions, multi-objective programming model are established and used LINGO software and Column-dropping Rule to achieve satisfied solution.**

Keywords: **Goal programming, Lexicographic Goal programming, multi-objective, LINGO software, Column-dropping Rule.**

1. INTRODUCTION

Multiple Objective optimizations technique is a type of optimization that handles problems with a set of objectives to be maximized or minimized.

In management, it is a system for determining the best solution when multiple goals exist. For example, a company may want to build a new production facility where taxes are low, customers are nearby, land is cheap, and potential employees are educated and well-trained [1, 23].

Many attempts have been made in the field of goal programming [17]. When the goals are based on a pre-emptive priority structure, then the lexicographical goal programming is utilized [5] and [10]. In many applications the decision-maker may not require to achieve his goals according to a priority rank structure, since the achievement of the goals having high priority levels might seriously affect the achievement of the goals with lower priority levels.

Modernization of management and scientific decision-making are key factors of the survival and development of enterprise. How to improve production management and how to conduct scientific decision- making are related to the life of enterprise. Modern scientific production management should be based on operation research theory and perspectives, arrange the various aspects of manpower, material and other means of production and production and marketing.

By rational allocation and full utilization of resources at the lowest possible cost, the best profits are obtained, to complete the scheduled economic goals. In the actual business management, decision-makers may also consider several objectives to achieve optimal: highest yield, lowest cost, best quality, most profit. It may also require multiple constraints, limited financial resources, limited duration; limited human and environmental compliance must meet the requirements [3, 7]. It can better take the co-ordinate relationship between the multiple objectives and conflicting constraints into account, and seek the more realistic solution requirements. Based on the actual situation, goal programming can possible meet various demands [9,10,13,25].

The paper is organized as follows: Introduction to Goal Programming Technique is presented in section Two. The Theory

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of Goal Programming is presented in section Three. Goal Programming Model is presented in section Four, whereas the illustrative example for Goal Programming problem is presented in section Five. Finally, section Six draws Conclusions.

2. INTRODUCTION TO GOAL PROGRAMMING TECHNIQUE

The Goal Programming (GP) technique has become a widely used approach in Operations Research (OR). Goal Programming model and its variants have been applied to solve large-scale multi-criteria decision-making problems. The Goal Programming technique was first used by Charnes and Cooper in 1960s.

The Goal Programming method is an improved method for solving multi objective problems. Goal Programming is one of the model which have been developed to dual with the multiple objectives decision-making problems.

The Goal Programming technique is an analytical framework that a decision maker can use to provide optimal solutions to multiple and conflicting objectives. Goal Programming is a special type of technique. This technique uses the simplex method for finding optimum solution of single dimensional or multi-dimensional objective functions with a given set of constraints which are expressed in linear form.

The computational procedure in goal programming is to select a set of solutions which satisfies the environmental constraints and providing a satisfactory goal, ranked in priority order. Low ordered goals are satisfied. If ordinal ranking of goals can be provided in terms of importance or contributions and all goal constraints are linear in nature, the solution of the portion can be obtained through Goal Programming.

Goal Programming method is not only a technique to minimize the sum of all deviations, but also a technique to minimize priority deviations as much as possible. The results of multi-objective problem solutions are affected by the decision of the manager or decision maker. Therefore, when there is a concession between goals, there will be deviations according to the decisions made. The direction and extent of these deviations play important roles in this type of problem.

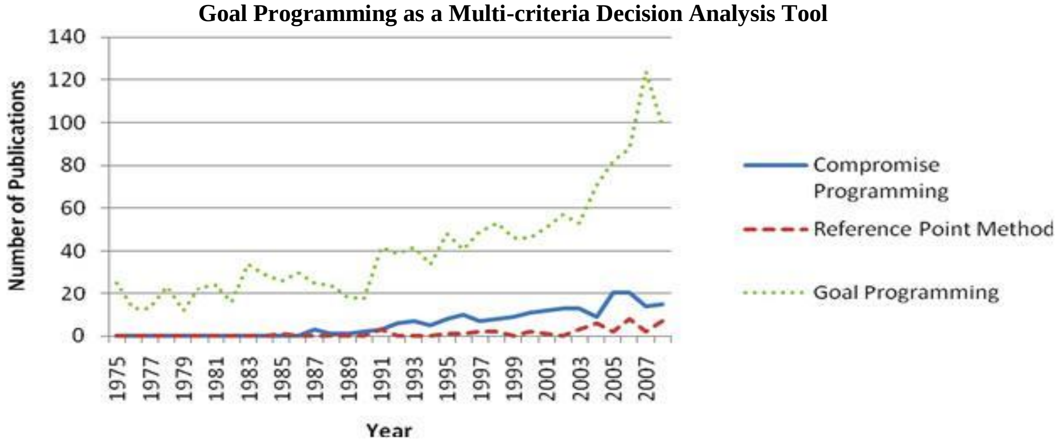
Goal Programming is used to manage a set of conflict objectives by minimize the deviations between the target values and the realized results.

The original objectives are re-formulated as a set of constraints with target values and two auxiliary variables. Two auxiliary variables are called positive deviation d^+ and negative deviation d^-, which represent the distance from this target value.

The objective of goal programming is to minimize the deviations hierarchically so that the goals of primary importance receive first priority attention; those of second importance receive second-priority attention, and so forth. Then, the goals of first priority are minimized in the phase. Using the obtained feasible solution result in the phrase, the goals of second priority are minimized, and so on.

Goal programming is one of the posteriori techniques, and most commonly method for solving multiple objective decision problems. (See Sunar and Kahraman (2001). Goal programming popularity from amongst the distance-based Multi-criteria Decision Maker techniques as described by Tamize and Jones (2010) demonstrates its continuous growth in recent years as represented below:

Goal Programming as a Multi-criteria Decision Analysis Tool



Source: Tamiz M, &D. F Jones (2010) Practical Goal Programming. International Series in Operations Research & Management Science. Springer New York http://www.springer.com/series/6161

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3. THE THEORY OF GOAL PROGRAMMING

Linear programming is able to handle the optimization problem of economics, management, and military aspects, but there are a lot of inadequacies:

(1) Linear programming requires the problems to be solved satisfy all constraints, but in practical problems, not all constraints need to strictly meet.

(2)Linear programming can only handle single-objective optimization problem, some sub-goals can only be translated as the constraint handling. In practical, objectives and constraints can be transformed into each other to deal with, not necessarily strictly differentiated.

(3)When dealing with problems, linear programming treats all constraints as equally important [4 12]. In practical, importance of each objective is different.

(4)Linear programming is to find the optimal solution, but many problems just seek to find a satisfactory solution.

The economics of the business is a multivariate and multi-objective network system, its production process includes many factors and characteristics of non-deterministic, and the behavior of enterprises tends to be the embodiment of the multi-objective [27]. Constraints in the linear programming model are stiff; the goal is single and set too dead, lack of flexibility. It does not fully take the diversity of the many variables in the production and business goals into account, so it is difficult to make an objective description of the production of enterprises and lacks practical value. In contrast, seeking a "satisfactory solution" in the target planning and multiple goals is more objective than linear programming and has greater practical significance [2,19,20].

3.1 Deviation variable:

(1) Deviation variable: concept of positive and negative deviations is introduced to represent the difference between decision-making value and target value.

d_i^+ —— Positive deviation variables; representing the part that decision value exceeds the target, set $d_i^+ \geq 0$.

d_i^- —— Negative deviation variables; representing the part that decision-making value is not reached the target, set $d_i^- \geq 0$.

In practice, when the target is determined, the decisions made there are three possibilities:

(i) Decision value exceeds the target value, expressed as $d_i^+ \geq 0$, $d_i^- = 0$.

(ii) Decision value dose not reach the target value, expressed as $d_i^+ = 0$, $d_i^- \geq 0$.

(iii) Decision value equals to the target value, expressed as $d_i^+ = 0$, $d_i^- = 0$.

(2) Absolute constraints and Goal constraints ：absolute constraint refers to constrains that must meet strict equality constraint and inequality constraints. Target constraint is unique in the goal programming to determine a target value to make decisions, allowing the existence of positive or negative deviation with the target value. For example, assume that a business plan profit is 6000, for the objective function:

Max $z = 300x_1 + 600x_2$

Can be transformed into:

$300x_1 + 600x_2 + d_i^- - d_i^+ = 6000$

3.2 Objective function:

In order to get the satisfactory solution which satisfies the system constraints and objective constraints, from the perspective of decision makers, determining their advantages and disadvantages should be based on the calculated value of the target deviation. Mathematical expression for the objective function of the goal programming model is:

Min $z = f(d_i^+, d_i^-)$

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It has the following three basic forms:

(1) Exactly reach the target, namely positive and negative deviation variables are as small as possible, the objective function is: Min $z= d_i^{+}+d_i^{-}$

(2) Does not exceed the target, namely positive deviation variable is as small as possible, the objective function is: Min $z =d_i^{+}$

(3) Exceed the target, namely the negative deviation variable is as small as possible, the objective function is: Min $z =d_i^{-}$

3.3 Pre-emptive Goal Programming (Lexicographic Goal Programming):

In many situations, however, a decision maker may not be able to determine precisely the relative importance of the goals. *I.e.* apply pre-emptive goal programming, the decision maker must rank his or her goals from the most important (goal 1) to least important (goal m). Preemptive goal programming procedure starts by concentrating on meeting the most important goal as closely as possible, before proceeding to the next higher goal, and so on to the least goal *i.e.* the objective functions are prioritized such that attainment of first goal is far more important than attainment of second goal which is far more important than attainment of third goal, etc, such that lower order goals are only achieved as long as they do not degrade the solution attained by higher priority goal**.**

When this is the case, pre emptive goal programming may prove to be a useful tool as introduced by Ijiri [6], and developed by many others.

The achievement function for the general preemptive GP model is given as

$$\text{Lexi Min } z = \sum_{I}^{k} p_i(d_i^- + d_i^+) \qquad (3.3.1)$$

Such that

$$\text{Min } z = \sum_{I}^{m} (w_i^- d_i^- + w_i^+ d_i^+) \qquad (3.3.2)$$

Such that equation,

$$\sum_{j}^{n} a_{ij}\, x_{ij} + d_i^- - d_i^+ = b_i \quad (i=1,2,\ldots m), \qquad (3.3.3) \quad \text{holds.}$$

The above is detailed in Orumie and Ebong [18].

Steps for the Preemptive Goal Programming algorithm is provided in Table 1.

Figure 1 depicts the flow chart of the overall algorithm.

Table 1. Preemptive Goal Programming Algorithm

S.No	STEP	ACTION
1	STEP :1	Embed the relevant data set. Set the first goal set as the current goal set.
2	STEP :2	Obtain a Linear Programming (LP) solution defining the current goal set as the objective function.
3	STEP :3	If the current goal set is the final goal set, **a.** set it equal to the LP objective function value obtained in Step 2, and STOP. Otherwise, go to Step 4.
4	STEP :4	If the current goal set is achieved or overachieved . **a.** set it equal to its aspiration level and add the constraint to the constraint set, Go to Step 5. **b.** Otherwise, if the value of the current goal set is underachieved, set the aspiration level of the current goal equal to the LP objective function value obtained in Step 2. Add this equation to the constraint set. Go to Step 5.
5	STEP :5	Set the next goal set of importance as the current goal set. Go to Step 2.

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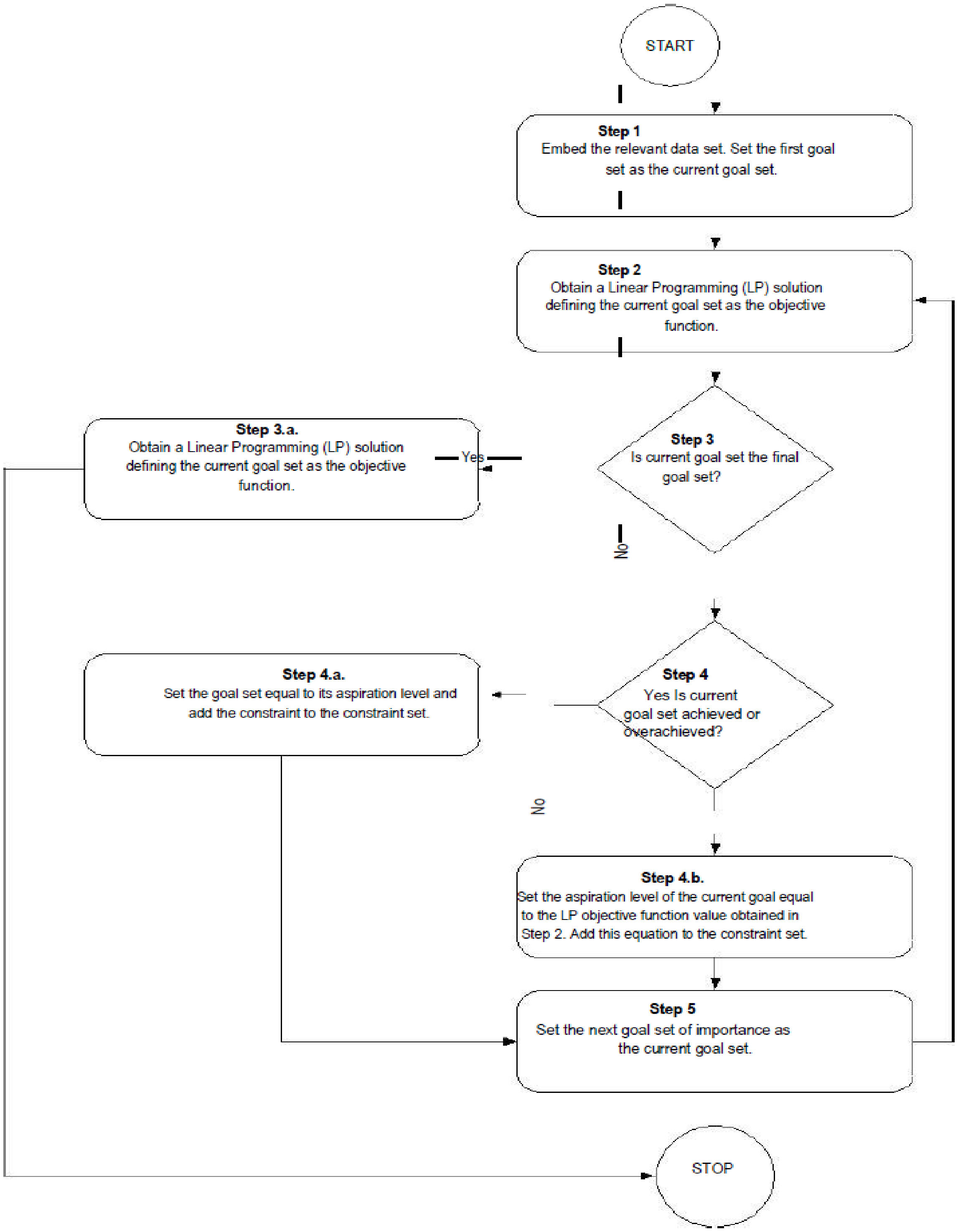

Figure 1. Flowchart of the Preemptive Goal Programming Algorithm

3.4 Priotized Goal Programming:

This is situation where both weighted and preemptive approaches are combined to form a model to a problem. This occurs when the goals can be categorised into groups where the goals within each group are of equal importance, but there are slight differences between the groups in their level of importance. In this kind of situation, weighted goal programming can be used within each group in turn while preemptive goal programming is being applied to deal with each group in order of importance. Each priority level (each group) has a number of unwanted deviations to be

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minimised. This means that minimisation of deviational variables placed in a higher priority level is assumed to be infinitely more important than that of deviational variables placed in a lower priority level (group). This is represented as in equations.

$$\text{Min } z = \sum_{i}^{m} (w_i^- d_i^- + w_i^+ d_i^+) \qquad (3.3.1)$$

Such that equations,

$$\sum_{j}^{n} a_{ij} x_{ij} + d_i^- - d_i^+ = b_i \quad (i=1,2,\dots m), \qquad (3.3.2)$$

$$x_{ij}, d_i^-, di^+ \geq 0, w_i > 0. \qquad (3.3.3)$$

$$(i=1,2,..m; j=1,2,..n) \qquad (3.3.4)$$

3.5 Solution of the goal programming model:

In general, the optimal solution for solving the goal programming model is impossible to make all objectives to achieve optimal. So, this so-called optimal solution is just the relative optimal solution, which makes the original goal to obtain better value under the premise of meeting some high-priority target. Satisfactory solution is also known as an acceptable solution.

(1) Acceptable Solution: If you seek the final solution, and the main objective $z_1^* = 0$ has been achieved, objective function value in some other level $z_k^* = a \neq 0$ $(2 \leq k \leq s)$, it illustrates that the goal of p_k has not been fully achieved, this final solution can be called an accepted solution. Acceptable solution is actually the satisfactory solution, which gives full consideration to higher-priority goals and weighs the other general targets.

(2) Unacceptable solution: If you seek the final solution, and the main objective has not been achieved, this final solution can be called an unaccepted solution. Now, the general approach is to relax the constraints or to reduce the main objective of a predetermined value appropriately, then to conduct the debugging and calculation of the model until fully realized the highest level of the main objectives.

4. GOAL PROGRAMMING MODEL

4.1 General form of the model:

In summary, goal programming model is consist of objective function, objective constraints, absolute constraints and variable non-negative constraints. The general form of the model is:

Objective Function:

$$\text{Min } z = \sum_{k=1}^{q} p_k \sum_{j=1}^{l} (w_{kj}^- d_j^- + w_{kj}^+ d_j^+) \qquad (j=1,2,\dots l), (k=1,2,\dots q)$$

Objective Constraints:

$$\sum_{j=1}^{n} c_{ij} x_{ij} + d_i^- - d_i^+ = g_i \ (i=1,2,\dots m),(j=1,2,..n)$$

Absolute Constraints:

$$\sum_{j=1}^{n} a_{ij} x_j = (\geq, \leq) b_i \quad (i=1,2,\dots m),(j=1,2,..n)$$

Non-negative Constraints:

$$x_j \geq 0 \ (j=1,2,\dots n) \ \ d_i^-, d_i^+ \geq 0 \ (i=1,2,\dots m).$$

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4.2 The steps of model-building:

In summary, following are the model-building steps:

(1) Determine the target and list goal constraints and absolute constraints according to the objectives and conditions proposed by the problem.

(2) According to the needs of decision makers, some or all absolute constraints be transferred into goal constraints, the method is to plus negative deviation variable and minus positive deviation variable in its left type of absolute constraint.

(3) Give the appropriate penalty coefficient p_k to all levels of the target (k=1, 2 …K).

(4) For the same priority objectives, its importance, give the appropriate weight coefficient according to its importance.

(5) Construct objective functions according to requirements of decision-makers.

5. THE ILLUSTRATIVE EXAMPLE FOR GOAL PROGRAMMING PROBLEMS

A new advertising agency with 10 employees, has received a contract to promote a new product. The agency can advertise by radio and television. The following table gives the number of people reached daily by each type of advertisement and the cost and labor requirements.

	RADIO	**TELEVISION**
Exposure (in millions of persons)/min	**4**	**8**
Cost (in thousands of dollars)/min	**8**	**24**
Assigned employees/min	**1**	**2**

The contract prohibits the advertising agency from using more than 5minutes of radio advertisement. Additionally, radio and television advertisements need to reach atleast 45 million people. The agency has a budget goal of $100,000 for the project. How many minutes of radio and television advertisement should the agency use?

5.1 Methods for Solving the Goal Programming Problem:

There are many methods for solving goal programming problems. We will introduce the software LINGO method, the Lexicographic method which includes the method of Column-dropping Rule.

5.1.1 LINGO Software sequential algorithm:

The algorithm is the order of priority, goal programming into a series of single objective of linear programming problem, while on a priority solution as the next priority constraints, and then the problem are solved by LINGO software. This approach for solving the model in illustrative example is following as,

Min $G_1=d_1^-$

Min $G_2 = d_2^+$

Min $G_3 = 2\, d_1^- + d_2^{+}$.

1. Find the first level goal:

Min = dminus1;

4*x1+8*x2+dminus1- dplus1=45;

8*x1+24*x2+dminus2 – dplus2=100;

x1+2*x2=10;

x1=6;

end

Calculated as follows:

Global optimum solution found.

Objective value: 10.000000

Total Solver iterations: 2

Variable	Value	Reduced cost
DMINUS1	0.000000	5.000000

2.Find the second level goal:

Min = dplus2;

4*x1+8*x2+dminus1- dplus1=45;

8*x1+24*x2+dminus2 – dplus2=100;

x1+2*x2=10;

x1=6;

dminus1=2;

end

Calculated as follows:

Global optimum solution found.

Objective value: 10.000000

Total Solver iterations: 2

Variable	Value	Reduced cost
DPLUS2	0.000000	5.000000

3.Find the third level goal:

Min = 2*dminus1+dplus2;

4*x1+8*x2+dminus1- dplus1=45;

8*x1+24*x2+dminus2 – dplus2=100;

x1+2*x2=10;

x1=6;

dminus1=2;

dplus2 =1.

end

Calculated as follows:

Global optimum solution found.

Objective value: 10.000000

Total Solver iterations: 2.000000

Variable	Value	Reduced cost
2*dminus1+dplus2	10.000000	0.000000
XI	5.000000	0.000000
X2	2.500000	0.000000
DMINUS1	2.000000	5.000000
DPLUS1	0.000000	0.000000
DMINUS2	0.000000	0.000000
DPLUS2	1.000000	0.000000

5.1.2 THE LEXICOGRAPHIC METHOD:

EXAMPLE: 1

A new advertising agency with 10 employees, has received a contract to promote a new product. The agency can advertise by radio and television. The following table gives the number of people reached daily by each type of advertisement and the cost and labor requirements.

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	RADIO	TELEVISION
Exposure (in millions of persons)/min	**4**	**8**
Cost (in thousands of dollars)/min	**8**	**24**
Assigned employees/min	**1**	**2**

The contract prohibits the advertising agency from using more than 5minutes of radio advertisement. Additionally, radio and television advertisements need to reach atleast 45 million people. The agency has a budget goal of \$100,000 for the project. How many minutes of radio and television advertisement should the agency use?

Solution:

The problem of example is solved by the Lexicographic method. Assume that the exposure goal has a higher priority.

Step 0:

$G_1 > G_2$

G_1: Minimize d_1^- (satisfy exposure goal)

G_2: Minimize d_2+ (satisfy budget goal)

Step 1:

Solve $LP_{1.}$

Minimize $G_1 = d_1^-$

Subject to

$4x_1+8x_2+d_1^- - d_1^+ = 45$; (Exposure goal)

$8x_1+24x_2+d_2^- - d_2^+ =100$;(Budget goal)

$x_1+2x_2 \leq 10$;(Personnel limit)

$x_1 \leq 6$;(Radio limit)

$x_1,x_2,d_1^-,d_1^+,d_2^-,d_2^+ \geq 0$.

		Cj	0	0	1	0	0	0
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1^-**	**S_1^+**	**S_2^-**	**S_2^+**
1	**S_1^-**	45	4	8	1	-1	0	0
0	**S_1^+**	100	8	**24**	0	0	1	-1
0	**S_2^-**	10	1	2	0	0	0	0
0	**S_2^+**	5	1	0	0	0	0	0
	Z_j-C_j	45	4	8	0	-1	0	0
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1^-**	**S_1^+**	**S_2^-**	**S_2^+**
1	**S_1^-**	35/3	4/3	0	1	-1	-1/3	1/3
0	**X_2**	25/6	1/3	1	0	0	1/24	-1/24
0	**S_2^-**	5/3	**1/3**	0	0	0	-1/12	1/12
0	**S_2^+**	5	1	0	0	0	0	0
	Z_j-C_j	35/3	4/3	0	0	-1	-1/3	1/3
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1^-**	**S_1^+**	**S_2^-**	**S_2^+**
1	**S_1^-**	5	0	0	1	-1	0	0
0	**X_2**	5/2	0	1	0	0	1/8	-1/8
0	**S_2^-**	5	1	0	0	0	-1/4	1/4
0	**S_2^+**	0	0	0	0	0	1/4	-1/4
	Z_j-C_j	5	0	0	0	-1	0	0

The optimum solution is X_1 = 5 minutes, X_2 = 2.5 minutes,

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d_1^- = 5 million people, with the remaining variables equal to zero. The solution shows that the exposure goal G_1 is violated by 5 million persons. The additional constraint to be added to the G_2^- problem is $d_1^- = 5$ (or, equivalent, $d_1^- \leq 5$).

Step 2:

The objective function of LP_2 is

Minimize $G_2 = d_2^+$

The constraints are the same as in step 1 plus the additional constraint $d_1^- = 5$.

In general, the additional constraint $d_1^- = 5$ can also be accounted for by substituting out d_1^- in the first constraint. The result is that the right hand- side of the exposure goal constraint will be changed from 45 to 40, thus reducing LP_2 to

Minimize $G_2 = d_2^+$

Subject to

$4x_1+8x_2 - d_1^+ \quad = 45$; (Exposure goal)

$8x_1+24x_2+d_2^- - d_2^+ = 100$;(Budget goal)

$x_1+2x_2 \leq 10$;(Personnel limit)

$x_1 \leq 6$;(Radio limit)

$x_1,x_2,d_1^-,d_1^+,d_2^-,d_2^+ \geq 0$.

The new formulation is one variable less than the one in LP_1, which is the general idea advanced by the column – dropping rule.

Actually, the optimization of LP_2 is not necessary in this problem, because the optimum solution to problem G_1 already yields $d_2^+ = 0$; that is, it is already optimum for LP_2. Such computational- saving opportunities should be taken advantage of whenever they arise during the course of implementing the Lexicographic method.

By using the Lexicographic method, In step 1, the optimum solution is X_1 = 5 minutes, X_2 = 2.5 minutes, d_1^- = 5 million people, with the remaining variables equal to zero. The solution shows that the exposure goal G_1 is violated by 5 million persons. The additional constraint to be added to the G_2^- problem is $d_1^- = 5$ (or, equivalent, $d_1^- \leq 5$).

In step 2, the optimization of LP_2 is not necessary in this problem, because the optimum solution to problem G_1 already yields $d_2^+ = 0$; that is, it is already optimum for LP_2.

5.1.3 Column – dropping Rule:

EXAMPLE: 2

In the same example, we show that a better solution for the problem of the weights method example and example1 can be obtained if the lexicographic method is used to optimize objectives rather than to satisfy goals. Later on, the same example is solved using the **column – dropping rule.**

The goals of example can be restated as

Priority 1: Maximize exposure (P_1)

Priority 2: Minimize cost (P_2)

Mathematically, the two objectives are given as

Maximize $P_1 = 4X_1 + 8X_2$ (Exposure)

Minimize $P_2 = 8X_1 + 24X_2$ (cost)

The specific goal limits for exposure and cost (= 45 and 100) in the weights method example and example 1 are removed, because we will allow the simplex method to determine these limits optimally.

The new problem can thus be stated as

Maximize $P_1 = 4X_1 + 8X_2$

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Minimize $P_2 = 8X_1 + 24x_2$

Subject to

$X_1 + 2X_2 \leq 10;$

$X_1 \leq 6;$

$X_1, X_2 \geq 0.$

We first solve the problem using the procedure introduced in example1.

Step1.

Solve LP_1

Maximize $P_1 = 4X_1 + 8X_2$

Subject to

$X_1 + 2X_2 \leq 10;$

$X_1 \leq 6;\ X_1, X_2 \geq 0.$

The optimum solution is $X_1 = 0$, $X_2 = 5$ with $P_1 = 40$, which shows that the most exposure we can get is 40 million persons.

Step 2:

Add the constraint $4X_1 + 8X_2 \geq 40$ to ensure that goal G is not degraded. Thus, we solve LP_2 as

Minimize $P_2 = 8X_1 + 24X_2$

Subject to

$X_1 + 2X_2 \leq 10;$

$X_1 \leq 6;$

$4X_1 + 8X_2 \geq 40$ (additional constraint);

$X_1, X_2 \geq 0.$

The optimum solution of LP_2 is P_2 = \$88,000, X_1 = 5 minutes, and X_2 = 2 minutes. It yields the same exposure (P_1 = 40 million people) but at a smaller cost than the one in example1, where we seek to satisfy rather than optimize the goals.

The same problem is solved now by using the **column dropping rule**. The rule calls for carrying the objective rows associated with all the goals in the simplex table, as we will show below.

LP_1 (Exposure Maximization):

The LP_1 simplex table carries both objective rows P_1 and P_2. The optimality condition applies to the P_1 -objective row only. The P_2 - row plays a passive role in LP_1 but must be updated (using the simplex row operations) with the rest of the simplex table in preparation for the optimization of LP_2.

LP_1 is solved in two iterations as follows:

Iterations:

		C_j	4	8	0	0
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1**	**S_2**
0	P_1	0	-4	-8	0	0
0	P_2	0	-8	-24	0	0
0	S_1	10	1	**2**	1	0
0	S_2	5	1	0	0	1
	Z_j-C_j	0	-4	-8	0	0
0	P_1	40	0	0	4	0
0	P_2	120	4	0	12	0
8	X_2	5	½	1	½	0
0	S_2	5	1	0	0	1
	Z_j-C_j	40	0	0	4	0

The last table yields the optimal solution $X_1 = 0$, $X_2 = 5$, and $P_1 = 40$.

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The **column-dropping** rule calls for eliminating any **non-basic** variable X_j with $Z_j - C_j \neq 0$ from the optimum tableau of LP_1 before LP_2 is optimized. The reason is that these variables, if left unchecked, could become positive in lower- priority optimization problems, which can degrade the quality of higher- priority solutions.

LP_2 (Cost Minimization):

The column-dropping rule eliminates S_1 (with $Z_j - C_j = 4$ in LP_1). We can see from the P_2 –row that if S is not eliminated, it will be the entering variable at the start of the P_2-iterations and will yield the optimum solution $X_1 = X_2 = 0$, which will degrade the optimum objective value of the P_1–problem from $P_1 = 40$ to $P_1 = 0$.

The P_2–problem is of the minimization type. Following the elimination of S_1, the variable X_1 with $Z_j - C_j = 4(>0)$ can improve the value of P_2. The following table shows the LP_2 iterations. The P_1 –row has been deleted because it serves no purpose in the optimization of LP_2.

		C_j	8	24	0	0	0
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1**	**S_2**	**S_3**
0	P_1	0	-4	-8	0	0	0
0	P_2	0	-8	-24	0	0	0
0	S_1	10	1	**2**	1	0	0
0	S_2	5	1	0	0	1	0
0	S_3	40	4	8	0	0	1
	Z_j-C_j	0	-4	-8	0	0	0
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1**	**S_2**	**S_3**
0	P_1	40	0	0	4	0	0
0	P_2	120	4	0	12	0	0
24	X_2	5	½	1	½	0	0
0	S_2	5	**1**	0	0	1	0
0	S_3	0	0	0	-4	0	1
	Z_j-C_j	120	4	0	12	0	0
0	P_1	40					
0	P_2	96	0	0	12	-4	0
24	X_2	2	0	1	½	-1/2	0
8	X_1	5	1	0	0	**1**	0
0	S_3	0	0	0	-4	0	1
	Z_j-C_j	88	0	0	12	-4	0

		C_j	**8**	**24**	**0**	**0**	**0**
C_B	**Y_B**	**X_B**	**X_1**	**X_2**	**S_1**	**S_2**	**S_3**
0	P_1	40	0	0	4	0	0
0	P_2	120	4	0	12	0	0
24	X_2	5	½	1	½	0	0
0	S_2	5	**1**	0	0	1	0
0	S_3	0	0	0	-4	0	1
	Z_j-C_j	120	4	0	12	0	0

The optimum solution ($X_1 = 5$, $X_2 = 2$) with a total exposure of $P_1 = 40$ and a total cost of $P_2 = 88$ is the same as obtained earlier.

By using the column dropping rule, In step 1, the optimum solution is $X_1 = 0$, $X_2 = 5$ with $P_1 = 40$, which shows that the most exposure we can get is 40 million persons.

In step 2, the optimum solution of LP_2 is $P_2 = \$88,000$, $X_1 = 5$ minutes, and $X_2 = 2$ minutes. It yields the same exposure ($P_1 = 40$ million people) but at a smaller cost than the one in example1, where we seek to satisfy rather than optimize the goals.

6. CONCLUSOINS

Goals are prioritized in some sense, and their level of aspiration is stated. An optimal solution is attained when all the goals are reached as close as possible to their aspiration level, while satisfying a set of constraints. In this paper, the goal programming techniques and the theory of goal programming which includes the concept of the deviation variables, the concept of objective function, the theory of Lexicographic Goal programming and prioritized Goal programming are introduced , then the general form of the model are established on the steps of the goal programming, and various methods for solving Goal Programming Problems such as the Lexicographic method including Column-dropping rule and LINGO Software method are also applied to solve the same model. LINGO Software is more convenient and practical by using of computers.

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